1

Ans 1: Here,

No. of red balls $= 4$

No. of blue balls $= 7$

$\therefore$ Total no. of balls $= 11$

i) $P(\text{2 red balls}) = \frac{4}{11} \times \frac{4}{11} = \frac{16}{121}$

ii) $P(\text{2 blue balls}) = \frac{7}{11} \times \frac{7}{11} = \frac{49}{121}$

iii) $P(\text{one red and one blue ball})$

$= P(\text{red and blue}) + P(\text{Blue and red})$

$= \left(\frac{4}{11} \times \frac{7}{11}\right) + \left(\frac{7}{11} \times \frac{4}{11}\right)$

$= \frac{2 \times 4 \times 7}{11 \times 11} = \frac{56}{121}$

Ans 2: Here, the sample space is :

$$S = \{(1,1), (1,2) \ldots, (6,5), (6,6)\}$$

$$\therefore n(S) = 36$$

Now P (sum of the no.s on the two dice is neither 9 nor 11)

$$= 1 - P(\text{sum of the no.s on the two dice is either 9 or 11})$$

$$= 1 - [P(\text{sum is } 9) + P(\text{sum is } 11) - P(\text{sum is 9 and 11})]$$

$$[\because P(A \cup B) = P(A) + P(B) - P(A \cap B)]$$

$$= 1 - \left[\frac{4}{36} + \frac{2}{36} - \frac{0}{36}\right]$$

$$= 1 - \frac{6}{36}$$

$= \frac{5}{6}$

Hence, P(sum of the numbers on the two dice is neither 9 nor 11)

$$= \frac{5}{6}$$

Ans 3: Let $I = \int \underset{II}{x} \, \underset{I}{\log 2x} \, dx$

Integrating by parts, we get,

$$I = \log 2x \int x \cdot dx - \int \left\{ \frac{d}{dx}(\log 2x) \int x \, dx \right\} dx$$

$$\Rightarrow I = (\log 2x) \times \frac{x^2}{2} - \int \frac{\cancel{2}}{2x} \times \frac{x^2}{\cancel{2}} \, dx$$

$$\Rightarrow I = \frac{x^2}{2} \log 2x - \frac{1}{2} \int x \, dx$$

$$= \frac{x^2}{2}\log 2x - \frac{1}{2} \times \frac{x^2}{2} + C$$

$$= \frac{x^2}{2}\log 2x - \frac{x^2}{4} + C$$

Hence

$$\int x \log 2x \, dx = \frac{x^2}{2}\log 2x - \frac{x^2}{4} + C.$$

Ans 4: Let $I = \int \cos 2x \cos 4x \, dx$

$$\Rightarrow I = \frac{1}{2}\int \cancel{2\cos 2x \cos 4x} \, dx = \frac{1}{2}\int 2\cos 2x \cos 4x . dx$$

$$= \frac{1}{2}\int \cos(2x+4x) + \cos(2x-4x) . dx$$

$$[\because 2\cos A \cos B = \cos(A+B) + \cos(A-B)]$$

$$= \frac{1}{2}\int \cos 6x \, dx + \frac{1}{2}\int \cos 2x . dx$$

12 ⑤

$= \frac{1}{2} \times \frac{\sin 6x}{6} + \frac{1}{2} \times \frac{\sin 2x}{2} + c$

$= \frac{\sin 2x}{4} + \frac{\sin 6x}{12} + c$

Hence

$$\int \cos 2x \cos 4x \, dx = \frac{\sin 2x}{4} + \frac{\sin 6x}{12} + c$$

Ans 5. Given that:

~~y = a cos (x~~

$y = a\cos(x+b)$ — ①

Differentiating w.r.t x, we get

$\frac{dy}{dx} = -a\sin(x+b)$

Differentiating again w.r.t x, we get

$$\frac{d^2y}{dx^2} = -a\cos(x+b).$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y \quad [\text{From } (1)]$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

This is the required differential eqⁿ. of the family of curves $y = a\cos(x+b)$

Ans 6: We have,

$$\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$$

$$\Rightarrow \sec^2 y \tan x\, dy = -\sec^2 x \tan y\, dx$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sec^2 x \tan y}{\sec^2 y \tan x}$$

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy = -\frac{\sec^2 x}{\tan x} dx$$

Integrating both sides, we get

$$\int \frac{\sec^2 y}{\tan y} dy = -\int \frac{\sec^2 x}{\tan x} dx \quad \text{——} \quad (1)$$

Putting $\tan y = a$ and $\tan x = b$,

we get, $\sec^2 y\, dy = da$ and $\sec^2 x\, dx = db$ resp.

Substituting these values in (1), we get

$$\int \frac{da}{a} = -\int \frac{db}{b}$$

$$\Rightarrow \log|a| = -\log|b| + \log C$$

$$\Rightarrow \log|a| + \log|b| = \log C$$

$$\Rightarrow \log|a.b| = \log C$$

$$\Rightarrow ab = C$$

$$\Rightarrow \underline{\tan x \cdot \tan y = C}$$

Ans 7: Given that α, β, γ are in A.P

$\Rightarrow (\beta - \alpha) = (\gamma - \beta) = d$ (say) ——— (1)

Let $\Delta = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} x-3 & x-4 & x \\ x-2 & x-3 & x \\ x-1 & x-2 & x \end{vmatrix} - \begin{vmatrix} x-3 & x-4 & \alpha \\ x-2 & x-3 & \beta \\ x-1 & x-2 & \gamma \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$ $\qquad$ $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Rightarrow \Delta = \begin{vmatrix} -1 & -1 & 0 \\ -1 & -1 & 0 \\ x-1 & x-2 & x \end{vmatrix} - \begin{vmatrix} -1 & -1 & \alpha-\beta \\ -1 & -1 & \beta-\gamma \\ x-1 & x-2 & \gamma \end{vmatrix}$$

$$\Rightarrow \Delta = 0 - \begin{vmatrix} -1 & -1 & -d \\ -1 & -1 & -d \\ x-1 & x-2 & \gamma \end{vmatrix}$$

[Using (1)]

[$\because$ R_1 & R_2 are identical in 1st term]

$= 0 - 0$ [$\because$ R_1 & R_2 are identical in 2nd term]

$= 0$

Hence,

$$\Delta = \begin{vmatrix} x-3 & x-4 & x-\alpha \\ x-2 & x-3 & x-\beta \\ x-1 & x-2 & x-\gamma \end{vmatrix} = 0$$

Ans 8. Here, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Hence

$$L.H.S = A^2 - 5A + 7I$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 = R.H.S$$

Now

$$A^2 - 5A + 7I = 0$$

Pre-multiplying both sides by A^{-1}, we get

$$A^{-1}A.A - 5A^{-1}.A + 7A^{-1}I = 0$$

$$\Rightarrow IA - 5I = -7A^{-1} \qquad [\because A^{-1}.A = I \text{ and } A^{-1}.I = A^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{7}(A - 5I)$$

$$\Rightarrow A^{-1} = -\frac{1}{7}\left[\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right]$$

$$\Rightarrow A^{-1} = -\frac{1}{7}\begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix}$$

3

Ans 9: We have

$$f(x) = x^2 - 4x + 3 \quad , \quad x \in [1,3]$$

The given f^n is a polynomial function,

Hence

i> $f(x)$ is continuous on $[1,3]$

ii> $f(x)$ is differentiable on $(1,3)$

iii> $f(1) = (1)^2 - 4(1) + 3 = 0$

and

$$f(3) = (3)^2 - 4(3) + 3 = 0$$

$\therefore f(1) = f(3)$

Hence, all the conditions for Rolle's theorem are satisfied.

Hence, there exists a pt. $x = c \in [1,3]$ such that $f'(c) = 0$

Now,

$f(x) = x^2 - 4x + 3$

$\Rightarrow f'(x) = 2x - 4$

Now $f'(c) = 0$

$\Rightarrow 2c - 4 = 0$

$\Rightarrow c = \frac{4}{2} = 2$

Hence, there exists a pt. $\underline{\underline{\overset{c}{\cancel{x}}}} = 2 \in [1,3]$ such that $f'(c) = 0$

Hence Rolle's theorem is verified for $f(x) = x^2 - 4x + 3$ on $[1,3]$

Ans 10: Given that:

$$y = Ae^{mx} + Be^{nx} \quad \text{——}$$

Differentiating w.r.t x, we get

$$\frac{dy}{dx} = mAe^{mx} + nBe^{nx} \quad \text{——}$$

Differentiating again w.r.t x, we get

$$\frac{d^2y}{dx^2} = m^2Ae^{mx} + n^2Be^{nx} \quad \text{——}$$

Hence,

$$\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$$

$$= m^2Ae^{mx} + n^2Be^{nx} - (m+n)(mAe^{mx} + nBe^{nx}) + mn(Ae^{mx} + Be^{nx})$$

[using ①, ② and ③]

$$= m^2Ae^{mx} + n^2Be^{nx} - m^2Ae^{mx} - mnBe^{nx} - mnAe^{mx}$$
$$- n^2Be^{nx} + mnAe^{mx} + mnBe^{nx}$$

$$= \underline{\underline{0}}$$

Hence proved.

1.1

Ans 11: We have :

S_1 : $p \rightarrow q$
S_2 : $\sim q$] → Hypothesis
S : $\sim p$: Conclusion

The truth table representing all the truth values of the given expressions is as follows.

(P.T.O)

Ans 12 H

p	q	HYPOTHESIS $p \to q$	HYPOTHESIS $\sim q$	CONCLUSION $\sim p$
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T ← critical Row

we observe that there is only one critical row and the truth value of the conclusion (i.e $\sim p$) in that row is also true hence,

the given argument is valid

Ans 12: Here,

$$f(x) = \sin(3x+2)$$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(3(x+h)+2) - \sin(3x+2)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(3x+2+3h) - \sin(3x+2)}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\left(3x+2+\frac{3h}{2}\right)\sin\left(\frac{3h}{2}\right)}{h}$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$$

$$= \lim_{h\to 0} \frac{\cos\left(3x+2+\frac{3}{2}h\right)\sin\left(\frac{3h}{2}\right)}{\frac{3h}{2}\times\frac{1}{3}}$$

$$= \lim_{h\to 0} \frac{\cos\left(3x+2+\frac{3}{2}h\right)}{1/3} \times \lim_{h\to 0} \frac{\sin\left(3h/2\right)}{3h/2}$$

$= 3\cos(3x+2) \times 1$

$= 3\cos(3x+2)$

$\therefore \frac{d}{dx}(\sin(3x+2)) = \underline{\underline{3\cos(3x+2)}}$

Ans 13: We have,

$$f(x) = \begin{cases} \dfrac{x^2-25}{x-5}, & x \neq 5 \\ k & x = 5 \end{cases}$$

Since $f(x)$ is given to be continuous at $x=5$, we have,

$$\lim_{x \to 5} f(x) = f(5)$$

$$\Rightarrow \lim_{x \to 5} \frac{x^2 - 25}{x - 5} = K$$

$$\Rightarrow \lim_{x \to 5} \frac{x^2 - 5^2}{x - 5} = K$$

$$\Rightarrow 2(5)^{2-1} = K \qquad \left[\because \lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}\right]$$

$$\Rightarrow 10 = K$$

Hence $K = 10$

14: Let $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$

$$\Rightarrow I = \int_0^{\pi/4} \log\left(1+\tan\left(\tfrac{\pi}{4}-x\right)\right) dx$$

$$\left[\because \int_0^a f(x).dx = \int_0^a f(a-x)\,dx\right.$$

$$\Rightarrow I = \int_0^{\pi/4} \log\left(1+\frac{\tan\pi/4 - \tan x}{1+\tan\pi/4\,\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left(1+\frac{1-\tan x}{1+\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left(\frac{1+\tan x + 1 - \tan x}{1+\tan x}\right) dx$$

$$\Rightarrow I = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right) dx \quad \text{---} ②$$

Adding ① and ②, we get

$$2I = \int_0^{\pi/4} \log(1+\tan x) + \log\left(\frac{2}{1+\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log\left((1+\tan x) \times \frac{2}{1+\tan x}\right) dx$$

$$= \int_0^{\pi/4} \log 2 \, dx = \log 2 \, [x]_0^{\pi/4}$$

$$= \frac{\pi}{4} \log 2$$

$$\Rightarrow I = \frac{\pi}{8} \log 2$$

Hence $\int_0^{\pi/4} \log(1+\tan x)\,dx = \frac{\pi}{8}\log 2$

Ans 15 Let $I = \int \frac{2x+1}{(x+2)(x-3)}\,dx$

Let $\frac{2x+1}{(x+2)(x-3)} = \frac{A}{x+2} + \frac{B}{x-3}$

$\Rightarrow 2x+1 = A(x-3) + B(x+2)$

$\Rightarrow 2x+1 = Ax + Bx - 3A + 2B$

comparing coefficient of x and the cons

we get

$A + B = 2$ and

$-3A + 2B = 1$

Solving these eqⁿs, we get-

$$A = \frac{3}{5}, \quad B = \frac{7}{5}$$

Hence,

$$I = \int \frac{2x+1}{(x+2)(x-3)}\,dx$$

$$= \frac{3}{5}\int \frac{dx}{x+2} + \frac{7}{5}\int \frac{dx}{x-3}$$

$$= \frac{3}{5}\log|x+2| + \frac{7}{5}\log|x-3| + C$$

$$= \log\left|(x+2)^{3/5}(x-3)^{7/5}\right| + C$$

Ans 16: (2nd option)

Here, $I = \int_0^3 (2x^2 + 3x + 5)\,dx$

Here $f(x) = 2x^2 + 3x + 5$, $a = 0$, $b = 3$

$\therefore \quad d = \frac{b-a}{n} = \frac{3}{n}$

Now

$f(0) = 2(0)^2 + 3(0) + 5$

$f(0+d) = 2d^2 + 3d + 5$

$f(0+2d) = 2 \times 2^2 d^2 + 3 \times 2d + 5$

$\vdots$

$f(0+(n-1)d) = 2(n-1)^2 d^2 + 3 \times (n-1)d + 5$

$\therefore f(0) + \dots + f(0+(n-1)d)$

$= 2d^2 \Sigma_1 (n-1)^2 + 3d\,\Sigma_1 (n-1) + 5n$

$$= \frac{2 \times 9}{n^2} \times \frac{n(n-1)(2n-1)}{6} + \frac{3 \times 3}{n} \times \frac{n(n-1)}{2} + 5n$$

$$= \frac{3}{n^2}(2n^3 - 3n^2 + n) + \frac{9(n-1)}{2} + 5n$$

$$= 3\left(2n - 3 + \frac{1}{n}\right) + \frac{9}{2}(n-1) + 5n \quad \text{---}(\ )$$

now, $I = \int_0^3 (2x^2 + 3x + 5)\, dx$

$$= \lim_{n\to\infty} d\,[f(0) + f(0+d) + \dots + f(0 + (n-1)d)]$$

$$= \lim_{n\to\infty} \frac{3}{n}\left[3\left(2n - 3 + \frac{1}{n}\right) + \frac{9}{2}(n-1) + 5n\right]$$

$$= \lim_{n\to\infty} 3\left[3\left(2 - \frac{3}{n} + \frac{1}{n^2}\right) + \frac{9}{2}\left(1 - \frac{1}{n}\right) + 5\right]$$

$$= 3\left[3(2) + \frac{9}{2}(1) + 5\right]$$

$$= 3\left[6 + \frac{9}{2} + 5\right]$$

$$= 3\left[\frac{12 + 9 + 10}{2}\right]$$

$$= \frac{3 \times 31}{2}$$

$$= \frac{93}{2}$$

Hence $\int_0^3 (2x^2 + 3x + 5)\,dx = \frac{93}{2}$

ROUGH

$\frac{2x^3}{3} + \frac{3x^2}{2}$

$18 + \frac{27}{2}$

$= \frac{36+27}{2}$

216
68
144

$2x + 2 + x($

$2x + 2 + x^3$

631

16399

378

$3962 \,2$

144

28

2241
86
794

Ans 17. Given curve:

$$x^2 = 4y \quad \text{—— (1)}$$

Let the given pt. be $P(-1, 2)$

Let (x, y) be any arbitrary point on (1)

The distance between (x, y) and P is given by

$$D^2 = (x+1)^2 + (y-2)^2$$

$$\Rightarrow D^2 = (x+1)^2 + \left(\frac{x^2}{4} - 2\right)^2 \quad [\text{ using (1) }]$$

Now $$\frac{dD^2}{dx} = 2(x+1) + 2\left(\frac{x^2}{4} - 2\right) \times \frac{1}{4} \times 2x$$

$$= 2(x+1) + x\left(\frac{x^2}{4} - 2\right)$$

Put $$\frac{d(D^2)}{dx} = 0$$

$$\Rightarrow 2(x+1) = -x\left(\frac{x^2 - 8}{4}\right)$$

$\Rightarrow 2x + 2 = \dfrac{-x^3 + 8x}{4}$

$\Rightarrow 8x + 8 = -x^3 + 8x$

$\Rightarrow x^3 = -8$

$\Rightarrow x = -2$

Now,

$$\frac{d^2(D^2)}{dx^2} = 2 + 0 + \frac{1}{4}(3x^2) - 2$$

$$= \frac{3}{4}x^2 > 0 \quad \forall x \in \mathbb{R}$$

Hence $x = -2$ is a pt. of minima.

i.e D^2 will be minimum when $x = -2$

$\Rightarrow D$ will be minimum when $x = -2$

Putting $x = -2$ in (1), we get

$4 = 4y$

$\Rightarrow y = 1$

Hence the point on the curve $x^2 = 4y$ which is nearest to the point $P(-1, 2)$ is $(-2, 1)$

Ans 18 Given that:

$$4x - 5y - 11z = 12$$
$$x - 3y + z = 1$$
$$2x + 3y - 7z = 2$$

The given system of eq^{n}s can be written in the form

$$X = A^{-1}B \quad \text{where}$$

$$A = \begin{bmatrix} 4 & -5 & -11 \\ 1 & -3 & 1 \\ 2 & 3 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix} \text{ and } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Now,

$|A| = 4(21-3) + 5(-7-2) - 11(3+6)$

$= 72 - 45 - 99$

$= -72$

Co-factors of A

$A_{11} = 18 \qquad A_{12} = +9 \qquad A_{13} = 9$

$A_{21} = -68 \qquad A_{22} = -6 \qquad A_{23} = -22$

$A_{31} = -38 \qquad A_{32} = -15 \qquad A_{33} = -7$

$$\therefore \ \text{Adj } A = \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \, Adj(A)$$

$$= \frac{-1}{72} \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix}$$

$$\therefore X = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{72} \begin{bmatrix} 18 & -68 & -38 \\ 9 & -6 & -15 \\ 9 & -22 & -7 \end{bmatrix} \begin{bmatrix} 12 \\ 1 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{-1}{72} \begin{bmatrix} (216 - 68 - 76) \\ (108 - 6 - 30) \\ (108 - 22 - 14) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{72} \begin{bmatrix} 72 \\ 72 \\ 72 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\Rightarrow x = -1, \quad y = -1, \quad z = -1$$

Ans 19. Let the two velocities be u and v.
Let α be the angle between u and v

$u = 12$ m/sec

$v = 20$ m/sec

$\alpha = 60^\circ$

Hence, the resultant R is given by

$$R = \sqrt{u^2 + v^2 + 2uv\cos\alpha}$$

$$= \sqrt{144 + 400 + (2 \times 12 \times 20 \times \cos 60^\circ)}$$

$$= \sqrt{544 + 240}$$

$$= \sqrt{784}$$

$$= 28 \text{ m/sec}$$

Let the resultant R make an angle θ with the direction of v; then

$$\tan\theta = \frac{u \sin\alpha}{v + u\cos\alpha}$$

$$= \frac{12 \times \frac{\sqrt{3}}{2}}{20 + 12 \times \frac{1}{2}}$$

$$= \frac{6\sqrt{3}}{26} = \frac{3\sqrt{3}}{13}.$$

$$\therefore \theta = \tan^{-1}\left(\frac{3\sqrt{3}}{13}\right)$$

Hence the resultant of two velocities 12 m/sec and 20 m/sec inclined at 60° to each other is 28 m/sec making an angle of $\tan^{-1}\left(\frac{3\sqrt{3}}{13}\right)$ with the direction of larger velocity.

Ans 20: Here,

$u = 30\ m/sec$

$a = 8\ m/sec^2$

i) we know that

$v = u + at$

Putting $u = 30\ m/sec$, $a = 8\ m/sec^2$ and $t = 7s$, we get

$v = 30 + 8(7)$

$\Rightarrow v = 30 + 56$

$\Rightarrow v = 86\ m/sec$

Hence, the velocity of the particle after 7 sec is 86 m/sec

ii) $s = ut + \frac{1}{2}at^2$

Here, $t = 6$ sec,

$\therefore s = 30(6) + \frac{1}{2} \times 8 \times 6 \times 6$

$= 180 + 144$

$= 324$ m

Hence, it will traverse 324 m in 6 seconds

iii) Here, $s = 100$ m

$\therefore s = ut + \frac{1}{2}at^2$

$\Rightarrow 100 = 30t + 4t^2$

$\Rightarrow 4t^2 + 30t - 100 = 0$

$\Rightarrow 2t^2 + 15t - 50 = 0$

$\Rightarrow 2t^2 + 20t - 5t - 50 = 0$

$\Rightarrow 2t(t+10) - 5(t+10) = 0$

$\Rightarrow (2t-5)(t+10) = 0$

$\Rightarrow t = \frac{5}{2}$ seconds ($t \neq -10$ $\because$ time can't be -ve)

$\therefore$ velocity of particle after $\frac{5}{2}$ seconds

$v = u + at$

$= 30 + 8\left(\frac{5}{2}\right)$

$= 50$ m/sec

Hence, velocity when it has traversed 100 m is 50 m/sec

Ans 21. Given vectors are :

$\vec{a} = -4\hat{i} - 6\hat{j} - 2\hat{k}$

$\vec{b} = -\hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{c} = -8\hat{i} - \hat{j} + \lambda\hat{k}$

Since $\vec{a}, \vec{b}, \vec{c}$ are coplanar,

$\therefore \quad [\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & \lambda \end{vmatrix} = 0$$

$\Rightarrow \quad -4(4\lambda + 3) + 6(-\lambda + 24) - 2(1 + 32) = 0$

$\Rightarrow \quad -16\lambda - 12 - 6\lambda + 144 - 66 = 0$

$\Rightarrow \quad -22\lambda = -144 + 66 + 12$

$\Rightarrow \quad -22\lambda = -144 + 78$

$\Rightarrow \quad -22\lambda = -66$

$\Rightarrow \quad \lambda = \dfrac{-66}{-22} = 3$

Hence $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\lambda = 3$

Ans 22: Here,

$$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$$
$$\vec{b} = \hat{i} + 3\hat{j} + \hat{k}$$
$$\vec{c} = \hat{i} + \hat{k}$$

$$\therefore \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

Hence, projection of $\vec{b} + \vec{c}$ on $\vec{a}$:

$$= (\vec{b} + \vec{c}) \cdot \hat{a}$$

$$= (\vec{b} + \vec{c}) \cdot \frac{\vec{a}}{|\vec{a}|}$$

$$= \frac{(2\hat{i} + 3\hat{j} + 2\hat{k}) \cdot (\hat{i} + 2\hat{j} + \hat{k})}{\sqrt{1 + 4 + 1}}$$

$$= \frac{2 + 6 + 2}{\sqrt{6}} = \frac{10}{\sqrt{6}}$$

Hence the projection of $\vec{b} + \vec{c}$ on $\vec{a}$ is $\frac{10}{\sqrt{6}}$ units

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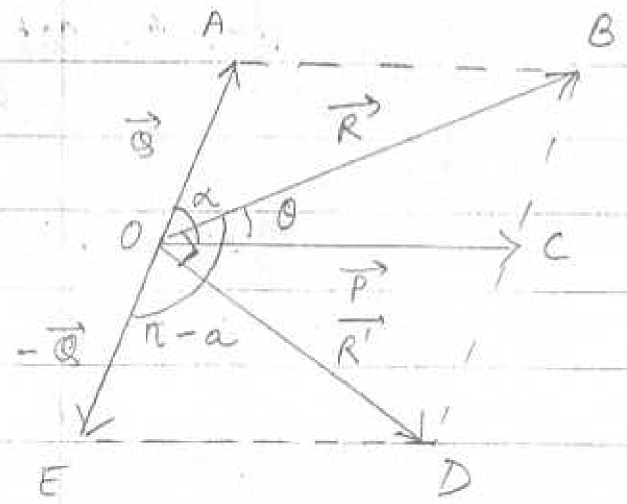
Ans 23: Let the two forces be P and Q. Let the angle between them be α.
Let their resultant be R and θ be the angle which R makes with P.



then, $\tan\theta = \dfrac{Q \sin\alpha}{P + Q\cos\alpha}$ —— (1).

Now, suppose the direction of Q is reversed, then the new resultant R' makes an angle of $\pi/2 - \theta$ with P.
Also Q makes an angle of $(\pi - \alpha)$ with P.

so, we have.

$$\tan\left(\frac{\pi}{2}-\theta\right) = \frac{q\sin(\pi-\alpha)}{p+q\cos(\pi-\alpha)}$$

$$\Rightarrow \cot\theta = \frac{q\sin\alpha}{p-q\cos\alpha} \quad \text{—} \ (\ )$$

Multiplying (1) and (2) we get

$$\tan\theta \cdot \cot\theta = \frac{q\sin\alpha}{p+q\cos\alpha} \times \frac{q\sin\alpha}{p-q\cos\alpha}$$

$$\Rightarrow p^2 - q^2\cos^2\alpha = q^2\sin^2\alpha$$

$$\Rightarrow p^2 = q^2\sin^2\alpha + q^2\cos^2\alpha$$

$$\Rightarrow p^2 = q^2(\sin^2\alpha + \cos^2\alpha)$$

$$\Rightarrow p^2 = q^2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow p = q$$

Hence proved.

Ans 24 : Given planes :

$$\vec{r}\cdot(2\hat{i}+\hat{j}+3\hat{k}) = 7 \quad \text{— (1)}$$

$$\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k}) = 9 \quad \text{— (2)}$$

Given point : $P(2,1,3)$

Now, eqⁿ of the plane passing through the intersection of the planes (1) and (2) is given by.

$$[\vec{r}\cdot(2\hat{i}+\hat{j}+3\hat{k})-7]+\lambda[\vec{r}\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9] = 0 \quad \text{— (i)}$$

Since the point with position vector $(2\hat{i}+\hat{j}+3\hat{k})$ lies on this plane, so it must satisfy its eqⁿ,

$$\therefore [(2\hat{i}+\hat{j}+3\hat{k})\cdot(2\hat{i}+\hat{j}+3\hat{k})-7]+\lambda[(2\hat{i}+\hat{j}+3\hat{k})\cdot(2\hat{i}+5\hat{j}+3\hat{k})-9] = 0$$

$$\Rightarrow (4+1+9-7)+\lambda(4+5+9-9) = 0$$

$$\Rightarrow 7+\lambda(9) = 0$$

$$\Rightarrow \lambda = -7,$$

Ans 24 : (continued)

Putting $\lambda = -7/9$ in (3), we get the required plane's eq^n as

$\vec{r}.[2\hat{i}+\hat{j}+3\hat{k}+\lambda(2\hat{i}+5\hat{j}+3\hat{k})] - 7 - 9\lambda = 0$

$\Rightarrow \vec{r}.[(2+2\lambda)\hat{i}+(1+5\lambda)\hat{j}+(3+3\lambda)\hat{k}] - 7 - 9\lambda = 0$

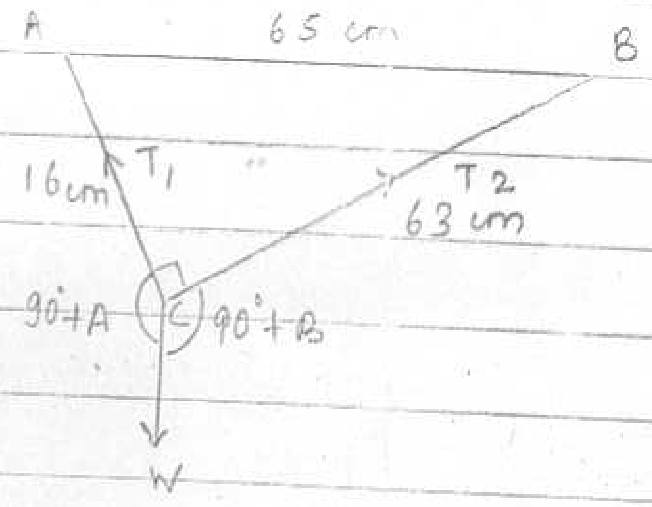
$\Rightarrow \vec{r}.\left[\left(2-\frac{14}{9}\right)\hat{i}+\left(1-\frac{35}{9}\right)\hat{j}+\left(3-\frac{21}{9}\right)\hat{k}\right] - \left[7-\frac{63}{9}\right] = 0$

$\Rightarrow \vec{r}.\left[\frac{4}{9}\hat{i} - \frac{26}{9}\hat{j} + \frac{6}{9}\hat{k}\right] - 0 = 0$

$\Rightarrow \vec{r}.(4\hat{i} - 26\hat{j} + 6\hat{k}) = 0$

This is the ... of ...

Ans 25 :



Let AC and BC be respectively strings of lengths 16 cm and 63 cm.

Let the weight be hung from C.

Now AB = 65 cm (Given)

We observe that

$$(16)^2 + (63)^2 = 256 + 3969 = 4225 = (65)^2$$

Hence ΔABC is right angled at C by Converse of Pythagoras theorem

Now, since the system is in equilibrium, therefore, at point C, we have

$$\frac{T_1}{\sin(90^\circ+B)} = \frac{T_2}{\sin(90^\circ+A)} = \frac{W}{\sin 90^\circ}$$

$$\Rightarrow \frac{T_1}{\cos B} = \frac{T_2}{\cos A} = W$$

$\Rightarrow T_1 = W\cos B$ and $T_2 = W\cos A$

$\Rightarrow T_1 = 5g \times \frac{63}{65}$ and $T_2 = 5g \times \frac{16}{65}$ [where $g = 9.8\,m/s^2$]

$\Rightarrow T_1 = \frac{5 \times 9.8 \times 63}{65}$ and $T_2 = \frac{5 \times 9.8 \times 16}{65}$

ROUGH

0.6990
0.9912
1.7993
1.8129
1.6766
0.6990
0.9912
1.2041
2.8913
1.8129
1.0814

(i) & (ii)

0

$\Rightarrow T_1 = 47.49\,N$ and $T_2 = 12.06\,N$

Hence the Tensions in the strings of length 16 cm and 63 cm are 47.49 N and 12.06 N respectively

1

Ans 26 : Let the equation of the sphere be

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

Since the points (0,0,0), (a,0,0), (0,b,0) and (0,0,c) lie on the sphere, they must satisfy its eq^n. So, we have

$$d = 0 \quad \text{—— (1)}$$

$$a^2 + 2ua = 0 \quad \text{—— (2)}$$

$$b^2 + 2vb = 0 \quad \text{—— (3)}$$

$$c^2 + 2wc = 0 \quad \text{—— (4)}$$

From 2, 3 and 4, we have

$u = -a/2$

$v = -b/2$

$w = -c/2$

Hence the required equation of the sphere is:

$$x^2 + y^2 + z^2 + 2\left(-\frac{a}{2}\right)x + 2\left(-\frac{b}{2}\right)y + 2\left(-\frac{c}{2}\right)z + 0 = 0$$

$$\Rightarrow x^2 + y^2 + z^2 - ax - by - cz = 0$$

of (i) & (ii)

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