Formulae of Integral Calculus

S.No	Chapter	Formulae	
1	**Integrals**	**1.1**	**Indefinite Integrals** $\int f(x)dx = F(x) + C$ where F(x) is the antiderivative of f(x) **Properties** $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$ $\int kf(x)dx = k\int f(x)dx$ for any real number k $\int [k_1f_1(x) + k_2f_2(x) + ... + k_nf_n(x)]dx =$ $k_1\int f_1(x)dx + k_2\int f_2(x)dx + ... + k_n\int f_n(x)dx$ where, $k_1, k_2 ... k_n$ are real numbers & $f_1, f_2, .. f_n$ are real functions
		1.2	**Some Standard Integrals** $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ $\int dx = x + C$ $\int \cos x\, dx = \sin x + C$ $\int \sin x\, dx = -\cos x + C$ $\int \sec^2 x\, dx = \tan x + C$ $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$ $\int \sec x \tan x\, dx = \sec x + C$ $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$ $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$ $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$ $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$ $\int \frac{dx}{1+x^2} = \cot^{-1} x + C$ $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C$

			$\int \frac{dx}{x\sqrt{x^2-1}} = -\operatorname{cosec}^{-1} x + C$ $\int e^x dx = e^x + C$ $\int a^x dx = \frac{a^x}{\log a} + C$ $\int \frac{1}{x} dx = \log\lvert x\rvert + C$
		1.3	**Integration by Partial Fractions** A rational function $\frac{P(x)}{Q(x)} = T(x) + \frac{P_1(x)}{Q(x)}$ if degree of P(x) > degree of Q(x) & $\frac{P_1(x)}{Q(x)}$ can be expressed as sum of partial fractions. $\frac{px+q}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b}$, $a \neq b$ $\frac{px+q}{(x-a)^2} = \frac{A}{x-a} + \frac{B}{(x-a)^2}$ $\frac{px^2+qx+r}{(x-a)(x-b)(x-c)} = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ $\frac{px^2+qx+r}{(x-a)^2(x-b)} = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$ $\frac{px^2+qx+r}{(x-a)(x^2+bx+c)} = \frac{A}{x-a} + \frac{Bx+C}{x^2+bx+c}$ where x^2+bx+c cannot be factorised further
		1.4	**Integration by substitution** A change in the variable of integration often reduces an integral to one of the fundamental integrals. Some standard integrals are: $\int \tan x\, dx = \log\lvert\sec x\rvert + C$ $\int \cot x\, dx = \log\lvert\sin x\rvert + C$ $\int \sec x\, dx = \log\lvert\sec x + \tan x\rvert + C$ $\int \operatorname{cosec} x\, dx = \log\lvert\operatorname{cosec} x - \cot x\rvert + C$ Standard substitutions x^2+a^2 substitute $x = a\tan\theta$ $\sqrt{x^2-a^2}$ substitute $x = a\sec\theta$ $\sqrt{a^2-x^2}$ substitute $x = a\sin\theta$ or $a\cos\theta$

		1.5	**Integral of some special functions** $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\log\left\|\frac{x-a}{x+a}\right\| + C$ $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}\log\left\|\frac{a+x}{a-x}\right\| + C$ $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$ $\int \frac{dx}{\sqrt{x^2-a^2}} = \log\left\|x+\sqrt{x^2-a^2}\right\| + C$ $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$ $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left\|x+\sqrt{x^2+a^2}\right\| + C$
		1.6	**Integration by parts** $\int f_1(x).f_2(x)dx = f_1(x)\int f_2(x)dx - \int\left[\frac{d}{dx}f_1(x).\int f_2(x)dx\right]dx$ where f_1 & f_2 are functions of x ILATE I- inverse trigonometric L- logarithmic A-algebra T-Trigonometric E-exponential , is used to identify the first function.
		1.7	**Some special type of integrals** $\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left\|x+\sqrt{x^2-a^2}\right\| + C$ $\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left\|x+\sqrt{x^2+a^2}\right\| + C$ $\int \sqrt{a^2-x^2}dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$
		1.8	**Area function** $A(x) = \int_a^x f(x)dx$, if x is a point in [a,b]

		1.9	**Fundamental Theorem of Integral Calculus** First fundamental theorem of integral calculus: If Area function, $A(x)=\int_a^x f(x)dx$ for all $x\geq a$, & f is continuous on [a,b].Then $A'(x)= f(x)$ for all $x\in [a, b]$. Second fundamental theorem of integral calculus: Let *f* be a continuous function of x in the closed interval [*a*, *b*] and let F be another function such that $\frac{d}{dx}F(x)=f(x)$ for all x in domain of f, then $\int_a^b f(x)dx=\left[F(x)+C\right]_a^b=F(b)-F(a)$
2	**Definite Integrals**	2.1	**Definite integral as limit of sum** $\int_a^b f(x)dx=(b-a)\lim_{n\to\infty}\frac{1}{n}\left[f(a)+f(a+h)+....+f(a+(n-1)h)\right]$ where $h=\frac{b-a}{n}$
		2.2	**Properties of Definite Integrals** $\int_a^b f(x)dx=\int_a^b f(t)dt$ $\int_a^b f(x)dx=-\int_b^a f(x)dx$ In particular, $\int_a^a f(x)dx=0$ $\int_a^b f(x)dx=\int_a^c f(x)dx+\int_c^b f(x)dx$ $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$ $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

			$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$ $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx, \text{if } f(2a-x) = f(x)$ $= 0 \quad , \text{if } f(2a-x) = -f(x)$ $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx, \text{if } f(-x) = f(x)$ $= 0 \quad , \text{if } f(-x) = -f(x)$
3	**Application of Integrals**	**3.1**	**Area of bounded region** The area of the region bounded by the curve y = f (x), x-axis and the lines x = a and x = b (b > a) is Area= $\int_a^b ydx = \int_a^b f(x)dx$ ▪ The area of the region enclosed between two curves $y = f(x)$, $y = g(x)$ and the lines $x = a$, $x = b$ is Area = $\int_a^b [f(x) - g(x)]dx$ where, f(x) > g(x) in [a,b] ▪ If f (x) ≥ g (x) in [a, c] and f (x) ≤ g (x) in [c,b], a < c < b, then Area = $\int_a^c [f(x) - g(x)]dx + \int_c^b [g(x) - f(x)]dx$