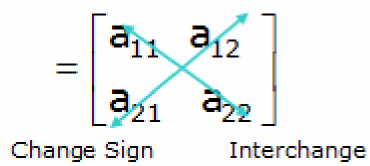
Formulae of Algebra

S.No	Chapter	Formula	
1	**Matrices**	1.1	**Types of matrices** $A = [a_{ij}]_{mxn}$ is a: ▪ Diagonal matrix if $a_{ij} = 0$, when $i \neq j$ ▪ Square matrix if m = n ▪ Row matrix if m = 1 ▪ Column matrix if n = 1 ▪ Scalar matrix if $a_{ij} = 0$, when $i \neq j$, $a_{ij} = k$, (some constant), when i = j ▪ Identity matrix if $a_{ij} = 1$, when i = j & $a_{ij} = 0$, when $i \neq j$ ▪ Zero matrix if $a_{ij} = 0$
		1.2	**Operations on matrices** ▪ Addition of Matrices: $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ then their sum $C = [c_{ij}]_{m\times n}$ $c_{ij} = a_{ij} + b_{ij}$ for $1 \le i \le m, 1 \le j \le n$ ▪ Scalar Multiplication: $A = [a_{ij}]_{m\times n}$ and k is a real number then $kA = [ka_{ij}]_{m\times n}$ ▪ Negative of a matrix :-A = (-1) A ▪ Difference of matrices: A – B = A + (-1) B ▪ Product of Matrices: If $A = [a_{ij}]_{m \times n}$ and $B = [b_{ik}]_{n \times p}$, then $AB = C = [c_{ik}]_{m \times p}$, where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{ij}$
		1.3	**Properties of matrices** ▪ -A = (-1) A (Negative of a matrix) ▪ A + B = B +A (Commutative Law of addition) ▪ A + (B+C) = (A + B)+C (Associative law of addition) ▪ k(A+B) = kA + kB (Multiplication by scalar)

			▪ $(k+L)A = kA + LA$ (Multiplication by scalar) ▪ $AB \neq BA$ in general ▪ $A(BC) = (AB)C$ (Associative law of multiplication) ▪ $A(B+C) = AB + AC$ (Distributive law) ▪ $(A+B)C = AC + BC$ (Distributive law)
		1.4	**Transpose of a Matrix** $A = \left[a_{ij}\right]_{mxn}$ then A' or $A^T = \left[a_{ji}\right]_{nxm}$
		1.5	**Properties of transpose of a matrix** ▪ $(A')' = A$ ▪ $(kA)' = kA'$ ▪ $(A+B)' = A'+B'$ ▪ $(AB)' = B'A'$
		1.6	**Inverse of a matrix** If AB = BA = I, where A & B are square matrices, then $B = A^{-1}$ or $A = B^{-1}$ & $(A^{-1})^{-1} = A$
		1.7	**Symmetric & Skew-symmetric matrices** ▪ $A = \left[a_{ij}\right]_{nxn}$ is symmetric if $A = A'$ i.e $a_{ij} = a_{ji}$ for all i and j ▪ $A = \left[a_{ij}\right]_{nxn}$ is skew symmetric if $A' = -A$ i.e If i=j, then $a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$ ▪ $A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$ Symmetric part of A $= \frac{1}{2}(A + A^T)$ Skew symmetric part of A $= \frac{1}{2}(A - A^T)$
		1.8	**Elementary operations of a matrix are as follows:** i. $R_i \leftrightarrow R_j$ or $C_i \leftrightarrow C_j$ ii. $R_i \rightarrow kR_i$ or $C_i \rightarrow kC_i$

			iii. $R_i \to R_i + kR_j$ or $C_i \to C_i + kC_j$
2	**Determinants**	2.1	**Determinant of order 2** If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then, $\|A\| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$
		2.2	**Determinant of order 3** If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then $\|A\| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ $= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$
		2.2	**Properties of determinants** For any square matrix A ▪ $\|A'\| = \|A\|$, where A' = transpose of A ▪ If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes. ▪ If any two rows (or columns) are identical or proportional then the value of determinant is 0. ▪ If each element of a row (or column) of a determinant is multiplied by a constant k, then its value gets multiplied by k. ▪ Multiplying a determinant by k means multiply each element of one row (or column) by k. ▪ If $A = [a_{ij}]_{nxn}$, then $\|k.A\| = k^n A$ ▪ If elements of a row (or column) can be expressed as sum of two or more elements then the determinant can be expressed as sum of two or more elements.

			▪ If to each element of a row (or column) of a determinant the equi-multiples of corresponding elements of other two rows or columns are added, then the value of determinant remains same. ▪ A has inverse if and only if A is non-singular ▪ Value of determinant is equal to the sum of product of element of a row (or a column) with its corresponding cofactors. ▪ If elements of one row (or column) are multiplied with cofactors of elements of any other row (or column), then their sum is 0. $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23} = 0$ ▪ , $\|AB\| = \|A\|\ \|B\|$,
		2.3	**Minors & Cofactors** ▪ Minor of an element a_{ij} of the determinant of matrix A is the determinant obtained by deleting the i^{th} row & j^{th} column denoted by M_{ij} ▪ Cofactor of a_{ij} is $A_{ij} = (-1)^{i+j} M_{ij}$
		2.5	**Adjoint & Inverse of a Matrix** ▪ If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then adj.A=  ▪ If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $\text{adj} A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$ where, A_{ij} are cofactors of a_{ij} ▪ $A^{-1} = \frac{1}{\|A\|}(\text{adj} A)$ ▪ $A(\text{adj } A) = (\text{adj } A) A = \|A\|\ I$ ▪ If A is a non singular matrix of order n then $\|\text{adj } A\| = \|A\|^{n-1}$

		2.6	**Area of a Triangle** Area of a triangle with vertices (x_1, y_1), (x_2,y_2) & (x_3,y_3) is $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$
		2.6	**Singular & non-singular matrices** ▪ Singular if $\lvert A \rvert = 0$ ▪ Non-singular if $\lvert A \rvert \neq 0$
		2.7	**Solution of Linear Equations** If $a_1x + b_1y + c_1z = d_1$ $a_2x + b_2y + c_2z = d_2$ $a_3x + b_3y + c_3z = d_3$ then these equations can be written as $AX = B$, where $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$ For $X = A^{-1}B$ if: ▪ $\lvert A \rvert \neq 0$, there exists a unique solution given by $X = A^{-1}B$. System of equations is consistent. ▪ $\lvert A \rvert = 0$ & (adj A)B≠0, there is no solution. System of equations is inconsistent. ▪ $\lvert A \rvert = 0$ & (adj A)B= 0, there exist infinitely many solutions. System of equations is consistent.