CBSE SAMPLE PAPER - III

CLASS XII MATHEMATICS

BLUE PRINT

S. No.	Topics	VSA	SA	LA	Total
1. (a)	Relations and Functions	-	4(1)	-	
(b)	Inverse Trigonometric Functions	2(2)	4(1)	-	10(4)
2. (a)	Matrics	1(1)	-	6(1)	
(b)	Determinants	2(2)	4(1)	-	13(5)
3. (a)	Continuity and differentiability	1(1)	12(3)	-	
(b)	Applications of derivatives	-	-	6(1)	
(c)	Integration	-	12(3)	-	
(d)	Application of integrals			6(1)	
(e)	Differential Equations	1(1)	-	6(1)	44(11)
4. (a)	Vectors	2(2)	4(1)	-	
(b)	3-dimensional Geometry	1(1)	4(1)	6(1)	17(6)
5.	Linear - Programming	-	-	6(1)	6(1)
6.	Probability	-	4(1)	6(1)	10(2)
	Total	10(10)	48(12)	42(7)	100(29)

SAMPLE PAPER - III

MATHEMATICS CLASS - XII

SECTION A

1. Write the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

2. Write the range of the principal branch of $\sec^{-1}(x)$ defined on the domain R-(-1, 1).

3. Find x if $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$.

4. If A is a square matrix of order 3 such that $|\text{adj A}| = 64$. Find $|A'|$

5. If A is a square matrix satisfying $A^2=1$, then what is the inverse of A?

6. If f(x) = sinx°, find $\frac{dy}{dx}$

7. What is the degree of the following differential equation?

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3 = x\left(\frac{d^3y}{dx^3}\right)^2$$

8. If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}|=3$, $|\vec{b}| = 4$ and $|\vec{a}\times\vec{b}| = 6$

10. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axes.

SECTION B

11. Show that the relation R in the set $A = \{x ; x \in Z, 0 \le x \le 12\}$ given by $R = \{(a, b) : |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

12. Solve for x : $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), 0<x<\frac{\pi}{2}$

OR

Show that : $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right] = \frac{x+y}{1-xy}$, $|x|<1$, $y>0$ $xy<1$

13. If none of a, b and c is zero, using properties of determinants.

prove that : $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3$

14. If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$, prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$

15. If $y=\left(x+\sqrt{x^2-1}\right)^m$, then show that $(x^2+1)\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-m^2y=0$

16. Find all the points of dicontinuity of the function f(x) = (x²) on [1, 2), where [.] denotes the greatest integer function.

OR

Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ w.r.t. $\cos^{-1}\left(\dfrac{1-x^2}{1+x^2}\right)$

17. Evaluate : $\displaystyle\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

OR

Evaluate : $\displaystyle\int x(\log x)^2 \cdot dx$

18. Evaluate : $\displaystyle\int \frac{x}{x^3-1}dx$

19. Using properties of definite integrals, evaluate.

$$\int_0^{\pi} \frac{x\,dx}{4-\cos^2 x}$$

20. The dot products of a vector with the vectors $\hat{i}-3\hat{k}$, $\hat{i}-2\hat{k}$ and $\hat{i}+\hat{j}+4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

21. Find the equation of plane passing through the point (1, 2, 1) and perpendicular to the line joining the points (1, 4, 2) and (2, 3, 5). Also, find the perpendicular distance of the plane from the origin.

OR

Find the equation of the perpendicular drawn from the point P(2, 4, -1) to the line $\dfrac{x+5}{1}=\dfrac{y+3}{4}=\dfrac{z-6}{-9}$.

22. A biased die is twice as likely to show an even number as an odd number. The die is rolled three times. If occurance of an even number is considered a success, then write the probability distribution of number of successes. Also find the mean number of successes.

SECTION C

23. Using matrices, solve the following system of equations :

$$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4;\ \frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0;\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2x\neq 0,\ y\neq 0,\ z\neq 0$$

24. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semivertical angle α, is $\frac{4}{27}\pi h^3 \tan^2\alpha$

OR

Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$ and $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

25. Find the area of the region: $\{(x,y) : 0 \leq y \leq x^2, 0 \leq y \leq x+2; 0 \leq x \leq 3\}$

26. Find the particular solution of the differential equation

$$(xdy-ydx)y\cdot\sin\left(\frac{y}{x}\right) = (ydx+xdy)x\cos\frac{y}{x},$$ given that $y=\pi$ when $x=3$.

27. Find the equation of the plane passing through the point (1, 1, 1) and containing the line

$\vec{r} = (-3\hat{i}+\hat{j}+5\hat{k}) + \lambda(3\hat{i}-\hat{j}+5\hat{k})$. Also, show that the plane contains the line

$\vec{r} = (-\hat{i}+2\hat{j}+5\hat{k}) + \lambda(\hat{i}-2\hat{j}-5\hat{k})$

28. A company sells two different products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit on each unit of A is Rs. 20 and on B is Rs. 15. How many units of A and B should be produced to maximise the profit. Form an L.P.P. and solve it graphically.

29. Two bags A and B contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

OR

In an examination, 10 questions of true - false type are asked. A student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true' and if it falls tails, he answers 'false'. Show that the probability that he answers at most 7 questions correctly is $\frac{121}{128}$.

MARKING SCHEME

MATHEMATICS CLASS - XII

SAMPLE PAPER III

SECTION A

1. $\dfrac{5\pi}{6}$

2. $\left[0, \dfrac{\pi}{2}\right) \cup \left(\dfrac{\pi}{2}, \pi\right]$

3. x=1

4. $|A'| = \pm 8$

5. $A^{-1}=A$

6. $\dfrac{\pi}{180}\cos x^{\circ}$

7. 2

8. $|\vec{a} \times \vec{b}|$

9. $\dfrac{\pi}{6}$

10. $< \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}} >$

SECTION B

11. (i) $\forall$ a $\in$ A, $|a-a|=0$ is divisible by 4. $\therefore$ R is reflexive --(i) ½

(ii) a, b $\in$ A, (a, b) $\in$ R $\Rightarrow |a-b|$ is divisible by 4.

$\Rightarrow |b-a|$ is divisible by 4 $\therefore$ R is symmetric --(ii) ½

(iii) a, b, c $\in$ A, (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow |a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4

$\therefore$ (a-b) and (b-c) are divisible by 4 and so

(a-b) + (b-c) = (a-c) is divisible by 4. Hence 1

$|a-c|$ is divisible by 4 $\Rightarrow$ (a, c) $\in$ R. Hence R is transitive

Hence R is an equivalence relation from (i), (ii) and (iii). ½

Set of all elements of A, related to 1 is {1, 5, 9} ½

12. Given equation can be written as

$$\tan^{-1}\left(\frac{2\sin x}{1-\sin^2 x}\right) = \tan^{-1}\left(\frac{2}{\cos x}\right),\ 0<x<\frac{\pi}{2}$$ 1½

$$\Rightarrow \frac{2\sin x}{\cos^2 x} = \frac{2}{\cos x} \Rightarrow \tan x = 1$$ 1½

$$\Rightarrow x=\frac{\pi}{4}$$ 1

OR

$$\text{LHS} = \tan\frac{1}{2}\left(2\tan^{-1}x+2\tan^{-1}y\right)$$ 1½

$$= \tan\left(\tan^{-1}x+\tan^{-1}y\right) = \tan\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$ 1½

$$= \frac{x+y}{1-xy}$$ 1

13. Given determinant can be writeen as

$$\Delta = \frac{1}{abc}\begin{vmatrix} -abc & ab(b+c) & ac(b+c) \\ ab(a+c) & -abc & bc(a+c) \\ ac(a+b) & bc(b+a) & -abc \end{vmatrix}$$ 1

$$\Delta = \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ab+ac \\ ab+bc & -ac & ab+bc \\ ac+bc & bc+ac & -ab \end{vmatrix} \begin{matrix} R_1 \to R_1+R_2+R_3 \\ = (ab+bc+ac) \end{matrix} \begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & ab+bc \\ ac+bc & bc+ac & -ab \end{vmatrix}$$ 1

$$\begin{matrix} C_2 \to C_2-C_1 \\ C_3 \to C_3-C_1 \end{matrix}\ \Delta = (ab+bc+ac)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & ab+bc \\ ac+bc & 0 & -(ab+bc+ac) \end{vmatrix}$$ 1

$$= (ab+bc+ac)^3$$ 1

14. Putting $x = \cos\alpha$ and $y = \cos\beta$ to get

$\sin\alpha + \sin\beta = a(\cos\alpha - \cos\beta) \Rightarrow \dfrac{2\sin\dfrac{\alpha+\beta}{2}\cos\left(\dfrac{\alpha-\beta}{2}\right)}{-2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}} = a$ 1

$\Rightarrow \cot\left(\dfrac{\alpha-\beta}{2}\right) = -a, \Rightarrow \alpha\text{-}\beta = 2\cot^{-1}(-a)$ or $\cos^{-1}x - \cos^{-1}y = 2\cot^{-1}(-a)$ 1

Differentiating to get $-\dfrac{1}{\sqrt{1-x^2}} + \dfrac{1}{\sqrt{1-y^2}}\dfrac{dy}{dx} = 0$ 1

$\Rightarrow \dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1\text{-}x^2}}$ 1]

15. Getting

$\dfrac{dy}{dx} = m\cdot\left(x+\sqrt{x^2+1}\right)^{m-1}\left(1+\dfrac{x}{\sqrt{x^2+1}}\right) = \dfrac{m\left(x+\sqrt{x^2+1}\right)^m}{\sqrt{x^2+1}} = \dfrac{m}{\sqrt{x^2+1}}\cdot y$ 1

$\Rightarrow \sqrt{x^2+1}\cdot\dfrac{dy}{dx} = my$ --(i) ½

$\therefore \sqrt{x^2+1}\cdot\dfrac{d^2y}{dx^2} + \dfrac{x}{\sqrt{x^2+1}}\cdot\dfrac{dy}{dx} = m\cdot\dfrac{dy}{dx}$ 1

$\Rightarrow (x^2+1)\dfrac{d^2y}{dx^2} + x\cdot\dfrac{dy}{dx} = m\sqrt{x^2+1}\dfrac{dy}{dx} = m\cdot my = m^2y$ (using i) 1

or $(x^2+1)\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} - m^2y = 0$ ½

16. $f(x) = [x^2],\ 1 \le x < 2 \Rightarrow f(x) = \begin{cases} 1, & 1 \le x < \sqrt{2} \\ 2, & \sqrt{2} \le x < \sqrt{3} \\ 3, & \sqrt{3} \le x < 2 \end{cases}$ 1

At $x = \sqrt{2}$, LHL =1, RHL = 2 $\therefore$ $x = \sqrt{2}$ is a discontinuity of f(x) 1½

At $x = \sqrt{3}$, LHL =2, RHL = 3 $\therefore$ $x = \sqrt{3}$ is also a discontinuity of(fx) 1

i.e. $\sqrt{2}, \sqrt{3}$ are two discontinuities in [1, 2) ½

OR

Let $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $z = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ ½

Put $x = \sin\theta$ to get

$y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$ and $z = 2\tan^{-1}x$ 1+½

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ and $\frac{dz}{dx} = \frac{2}{1+x^2}$ 1

$\Rightarrow \frac{dy}{dz} = \frac{1+x^2}{\sqrt{1-x^2}}$ 1

17. $I = \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(b-a)}\int \frac{\sin[(x-a)-(x-b)]}{\cos(x-a)\cos(x-b)}dx$ 1

$= \frac{1}{\sin(b-a)}\int\left[\frac{\sin(x-a)\cos(x-b)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-a)\sin(x-b)}{\cos(x-a)\cos(x-b)}\right]dx$ 1

$= \frac{1}{\sin(b-a)}\int[\tan(x-a)-\tan(x-b)]dx$ 1

$= \frac{1}{\sin(b-a)}\cdot\left[\log|\sec(x-a)| - \log|\sec(x-b)|\right] + c$ 1

OR

$I = \int(\log x)^2\cdot x\,dx = (\log x)^2\cdot\frac{x^2}{2} - \int 2\cdot\frac{\log x}{x}\cdot\frac{x^2}{2}dx$ 1½

$= \frac{x^2}{2}\cdot(\log x)^2 - \log x\cdot\frac{x^2}{2} + \int\frac{1}{x}\cdot\frac{x^2}{2}dx$ 1½

$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + c$ or $\frac{x^2}{2}\cdot\left[(\log x)^2 - \log x + \frac{1}{2}\right] + c$ 1

18. $\frac{x}{x^3-1} = \frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \Rightarrow x = A(x^2+x+1) + (Bx+C)(x-1)$ ½

$\Rightarrow$ A+B = 0, A-B+C = 1 and A-C = 0 $\Rightarrow A = \frac{1}{3},\ b = -\frac{1}{3},\ c = \frac{1}{3}$ 1

$\therefore I = \int\frac{x}{x^3-1}dx = \frac{1}{3}\int\frac{1}{x-1}dx - \frac{1}{3}\int\frac{x-1}{x^2+x+1}dx$ 1

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\int \frac{2x+1}{x^2+x+1}dx + \frac{1}{2}\int \frac{1}{x^2+x+1}dx$$ 1

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\log|x^2+x+1| + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)+\left(\sqrt{\frac{3}{2}}\right)^2}dx$$

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\log|x^2+x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}} + c$$ ½

19. $$I=\int_0^{\pi} \frac{x\,dx}{4-\cos^2 x} = \int_0^{\pi} \frac{(\pi-x)dx}{4-\cos^2(\pi-x)} = \int_0^{\pi} \frac{(\pi-x)dx}{4-\cos^2 x}$$ 1

$$\therefore\ 2I = \pi\int_0^{\pi} \frac{1}{4-\cos^2 x}dx = 2\pi\int_0^{\pi/2} \frac{\sec^2 x}{4-\tan^2 x+3}$$ 1

$$I = \frac{\pi}{4}\int_0^{\infty}\frac{dt}{t^2+3/4},\ \tan x = t \Rightarrow I = \frac{\pi}{4}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}}\tan^{-1}\left.\frac{2t}{\sqrt{3}}\right]_0^{\infty}$$ 1

$$I = \frac{\pi}{2\sqrt{3}}\cdot\frac{\pi}{2} = \frac{\pi^2}{4\sqrt{3}}$$ 1

20. Let the required vector be $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore\ \vec{a}\cdot(\hat{i}-3\hat{k}) = 0 \Rightarrow x-3z = 0$ --(i) 1

$\vec{a}\cdot(\hat{i}-2k) = 5 \Rightarrow x-2z = 5$ --(ii) ½

$\vec{a}\cdot(\hat{i}+\hat{j}+4\hat{k}) = 8 \Rightarrow x+y+4z = 8$ --(iii) ½

solving (i) and (ii) to get x=15, z=5 1

Putting in (iii) to get y = -27 ½

$\vec{a} = 15\hat{i}-27\hat{j}+5\hat{k}$ ½

21. Here $\vec{a} = \hat{i}+2\hat{j}+\hat{k}$ and $\vec{n}=(2-1)\hat{i}+(3-4)\hat{j}+(5-2)\hat{k}$

$= \hat{i}-\hat{j}+3\hat{k}$ 1

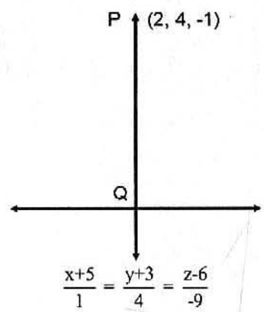
$\therefore$ equation of plane is $\vec{r}\cdot(\hat{i}-\hat{j}+3\hat{k}) = (\hat{i}+2\hat{j}+\hat{k})\cdot(\hat{i}-\hat{j}+3\hat{k})=2$ 1+1

or x-y+3z-2 = 0

Distance from origin $= \dfrac{2}{\sqrt{1+1+9}} = \dfrac{2}{\sqrt{11}}$ or $\dfrac{2\sqrt{11}}{11}$ units 1

OR

Any point on the given line is, $(\lambda-5,\ 4\lambda\text{-}3,\ \text{-}9\lambda\text{+}6)$ for some value of λ, this point is Q, such that PQ is $\perp$ to the line 1



$\Rightarrow\ (\lambda\text{-}7)1+(4\lambda\text{-}7)4+(\text{-}9\lambda\text{+}7)(\text{-}9) = 0 \Rightarrow \lambda=1$ 1½

$\therefore$ Q is (-4, 1, -3) and equation of line PQ is

$$\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$$ 1

22. Getting P(odd number) $= \dfrac{1}{3}$, P(even number) $= \dfrac{2}{3}$ ½

Let X be the random variable "getting an even number"

$\therefore$ X	0	1	2	3	½
P(X)	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$	1½
$X \cdot P(X)$	0	$\frac{6}{27}$	$\frac{24}{27}$	$\frac{24}{27}$	½

Mean $= \sum XP(X) = \dfrac{54}{27} = 2$ 1

23. Given equation can be written as

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \text{ or } A \cdot X = B$$ ½

$|A| = 1(4)+1(5)+1(1) = 10 \neq 0 \ \therefore\ X = A^{-1} \cdot B$ 1

cofactors are :

$A_{11}=4,\ A_{12}=-5,\ A_{13}=1$

$A_{21}=2,\ A_{22}=0,\ A_{23}=-2$ 2

$A_{31}=2,\ A_{32}=5,\ A_{33}=3$

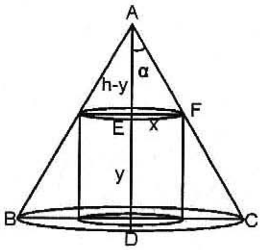
$$\therefore\ A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & 2 & 3 \end{pmatrix}$$ 1

$$\begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$ 1

$\Rightarrow\ x=\frac{1}{2},\ y=-1,\ z=1$ ½

24. Let the radius of inscribed cylinder be x and its height be y

$\therefore$ Volume (v) $= \pi x^2 y$ ½



$= \pi (h-y)^2 \tan^2\alpha \cdot y$

$= \pi \tan^2\alpha \left[h^2y - 2hy^2 + y^3\right]$ 1

$\frac{dv}{dy} = \pi \tan^2\alpha \left[h^2 - 4hy + 3y^2\right]$ 1

$\frac{dv}{dy} = 0 \Rightarrow 3y^2 - 4hy + h^2 = 0$ or $3(y-h)(3y-h) = 0 \Rightarrow y=h,\ y=h/3$ 1½

since y=h is not possible $\therefore\ y=h/3$ is the only point

$\frac{d^2v}{dy^2} = 6y - 4h = 6\left(h/3\right) - 4h = -2h < 0\ \therefore\ y=\frac{h}{3}$ is a maxima 1

$= \frac{4}{27}\pi h^3 \tan^2\alpha$

OR

$\frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta = a\theta\cos\theta$ 1

$\frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta = a\theta\sin\theta$ 1

$\Rightarrow \frac{dy}{dx} = \tan\theta \ \therefore$ slope of normal = $-\cot\theta$ 1

$\therefore$ Equation of normal is

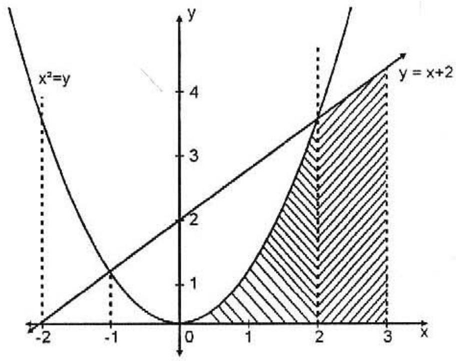
$$y\text{-}a(\sin\theta - \theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}[x\text{-}a(\cos\theta + \theta\sin\theta)]$$ 1

Simplifying to get $x\cos\theta + y\sin\theta - a = 0$

Length of perpendicular from orgin = $\frac{|a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a|$ (constant 1

25. For correct figure getting points of intersection as x=-1, x=2 ½

Required area = $\int_0^2 x^2dx + \int_2^3 (x+2)dx$ 2



$$\left[\frac{x^3}{3}\right]_0^2 + \left[\frac{(x+2)^2}{2}\right]_2^3$$ 1½

$$\frac{8}{3} + \frac{25}{2} - 8 = \frac{43}{6} \text{ sq.U}$$ 1

26. Given differential equation can be written as

$$\left(xy\frac{dy}{dx} - y^2\right)\sin\left(\frac{y}{x}\right) = \left(xy + x^2\frac{dy}{dx}\right)\cos\left(y/x\right) \quad \text{--(i)}$$ ½

Putting $\frac{y}{x} = v$ or $y = vx$ gives $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\therefore$ (i) becom $v\sin v\left(v+x\frac{dv}{dx}\right) - v^2\sin v = v\cos v + \left(v+x\frac{dv}{dx}\right)\cos v$ 1

$$\Rightarrow (vx\sin v - x\cos v)\frac{dv}{dx} = 2v\cos v$$

$$\Rightarrow -\int\frac{v\sin v - \cos v}{v\cos v}dv = -\int\frac{2}{x}dx \Rightarrow \log|v\cos v| = -2\log x + \log c$$ 1+1

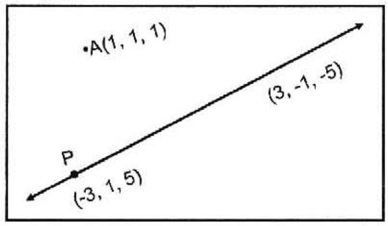
$$\Rightarrow x^2 \cdot v \cdot \cos v = c \quad \Rightarrow xy\cos y/x = c$$ ½

$x=3,\ y=\pi$ gives $c = \frac{3\pi}{2}$ ½

$\Rightarrow$ solution is $2xy \cos y/x = 3\pi$ ½

27. Let the given point be A (1, 1, 1) and the point on the line is P(-3, 1, 5)

$\therefore$ $\overrightarrow{AP} = -4\hat{i} + 4\hat{k}$ 1



$\therefore$ The vector $\perp$ to the plane is

$(-4\hat{i} + 4\hat{k}) \times (3\hat{i}-\hat{j}-5\hat{k}) = 4\hat{i}-8\hat{j}+4\hat{k}$ or $\hat{i}-2\hat{j}+\hat{k}$ 1½

$\therefore$ Equation of plane is

$\vec{r} \cdot (\hat{i}-2\hat{j}+\hat{k}) = (\hat{i}+\hat{j}+\hat{k}) \cdot (\hat{i}-2\hat{j}+\hat{k}) = 0$ --(i) 1½

or $x-2y+z = 0$

Now, since $(\hat{i}-2\hat{j}-5\hat{k}) \cdot (\hat{i}-2\hat{j}+\hat{k}) = 1+4-5 = 0$

$\therefore$ The line $\vec{r} = (-\hat{i}+2\hat{j}+5\hat{k}) + \lambda(\hat{i}-2\hat{j}-5\hat{k})$ is parallel to the plane 1

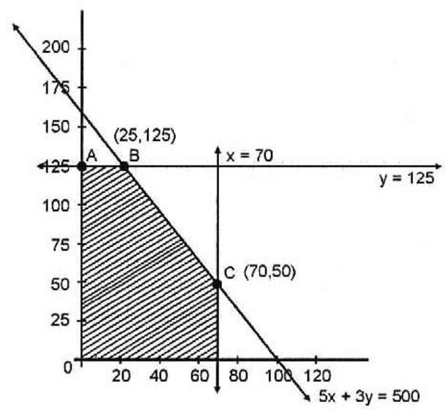
Also, the point (-1, 2, 5) satisfies the equation of plane

as $(-1-4+5) = 0 \Rightarrow$ point lines on plane 1

hence the plane contains the line.

28. Let x be the number of units of A and y of B, which are produced

$\therefore$ LPP is Maximise $z = 20x+15y$ 1



Subject to $5x+3y \leq 500$

$x \leq 70$ 1½

$y \leq 125$

$x \geq 0,\ y \geq 0$

2

Getting vertices of feasible region as :

A(0, 125), B(25, 125), C(70, 50), D(70, 0) ½

Maximum Profit = Rs. 2375 at B

$\therefore$ Number of Units of A = 25 1

Number of Units of B = 125

29. Let the events are defined as :

E_1 : 2 white balls are transferred from A to B

E_2 : 2 black balls are transferred

E_3 : 1 white and 1 black ball is transffered 1

A : 1 black ball is drawn from B

$$P(E_1) = \frac{4c_2}{7c_2} = \frac{4.3}{7.6} = \frac{2}{7},\ P(E_2) = \frac{3c_2}{7c_2} = \frac{3.2}{7.6} = \frac{1}{7},\ P(E_3) = \frac{4c_1 \cdot 3c_1}{7c_2} = \frac{4}{7} \quad 1\tfrac{1}{2}$$

$$P(A/E_1) = \frac{2}{6} = \frac{1}{3},\ P(A/E_2) = \frac{4}{6} = \frac{2}{3},\ P(A/E_3) = \frac{3}{6} = \frac{1}{2} \quad 1\tfrac{1}{2}$$

$$P(E_3/A) = \frac{P(E_3) \cdot P(A/E_3)}{P(E_1)\, P(A/E_1) + P(E_2)\, P(A/E_2) + P(E_3)\, P(A/E_3)} \quad \tfrac{1}{2}$$

$$= \frac{\frac{4}{7} \times \frac{1}{2}}{\frac{2}{7} \cdot \frac{1}{2} + \frac{1}{7} \cdot \frac{2}{3} + \frac{4}{7} \cdot \frac{1}{2}} \quad \tfrac{1}{2}$$

$$= \frac{3}{5} \quad 1$$

OR

$$\text{P(answer is true)} = \frac{1}{2}$$

$$\text{P(answer is false)} = \frac{1}{2} \quad 1$$

P(at most 7 correct) = 1-{P(8) + P(9) + P(10)} 2

(where P(8) etc means probability of 8 correct answers)

$$= 1 - \left\{ {}^{10}C_8 \left(\frac{1}{2}\right)^8 \cdot \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \right\} \quad 1$$

$$= 1 - \left\{ {}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0 \right\} \left(\frac{1}{2}\right)^{10} \quad 1$$

$$= 1 - \{45 + 10 + 1\} \frac{1}{1024}$$

$$= 1 - \frac{56}{1024} = 1 - \frac{7}{128} = \frac{121}{128} \quad 1$$